

Exchangeability Theorems as Categorical Limits of Probabilistic and Quantum Processes



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You wanna know how to rhyme?

You better learn how to add.

— Yasiin Bey

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I arrived in Oxford in October 2020 and in the first week met more people than I had in the preceding six months, alongside beginning a research degree in *theoretical computer science*, which meant little to me at the time, and having to move flat because of a cockroach infestation. Half of that year was spent in London playing too much online chess and learning about sheaves, and when it came to an end I owed what remained of my mental health to Nicolas Garraud, Paul Micoud, Brianna Yang and Lindiwe Makgalemele.

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²Steve S (<https://math.stackexchange.com/users/1557600/steve-s>). *The Algebraic Tensor Product of C^* -Algebras as an Operator System under Different Norms*. Mathematics Stack Exchange. Feb. 2025. eprint: <https://math.stackexchange.com/q/5031395>.

Roland, thank you for feeding me, in all senses of the word, in a time when I would have struggled to feed myself.

Another source of joy in this period was the radio show I cohosted with Ben Caspi, *Ben Nedis: The Tallest Mountain In Variety Town*, on the Oxide radio station³. Once a week we would chat and try to provide ‘genre whiplash’ for our listeners, of which there were few. It was really, really good fun, and to Ben, I am grateful for your company and your music.

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³For as long as it remains hosted online by Oxide, you may find the final episode here: <https://www.nedsummers.com/BenNedisFinalEp>

⁴Not to be mistaken for Clovis, the human man, earlier thanked in these acknowledgements

one month before when I intended to finish this degree (an insane decision on my part). CNM indeed.

I moved to Berlin with the hope of building a creative community. I have read poems and told stories on stage and in smoking areas around the city. Berlin is also blessed with bookshops and cinemas and I have read and watched more than any other year of my life, but far beyond these exercises in performance and consumption, the greatest creative addition to my Berlin has been Sara Bresciani. Sara, here we stand at the perfect moment of our lives to have met each other. It has been my privilege to be part of your Berlin, to have you be part of mine and to be able to count on your company in whatever follows.

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⁵Not to be mistaken for Stanley, the human man, earlier thanked in these acknowledgements.

⁶Luke Summers. 'Echoes of the Shoah: British Jewry and the Bosnian War'. In: *Holocaust Studies* (Aug. 2024), pp. 1–22.